AUTOMATION OF THE COMPUTING PROCESS IN SOLVING PROBLEMS OF HEAT CONDUCTION BY THE FINITE-ELEMENT METHOD FOR HEAT-INSULATING STRUCTURES WITH IRREGULAR BOUNDARIES

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The article examines questions of the automation of the finite-element solution of heat-conduction problems with phase transformations and mobile boundaries. The main principles of devising an algorithm and the results of a numerical experiment are given.

In the solution of non-steady-state heat-conduction problems in structures with irregular boundaries, complex functional dependences in regard to boundary conditions and thermophysical characteristics of the material (or set of materials), the finite-element method [1-3] is widely used nowadays. As a rule, the practical computer realization of this method requires an enormous amount of initial information. The specific traits of the work of heat insulation due to structural and phase transformations, the formation of various mobile zones (carburization, decomposition) and removal of material increase even more the flow of information because it is necessary to construct finite-element networks in time. Therefore, in the solution of this class of problems, problems of automating the finite-element procedure are very topical and in themselves deserve close attention. The literature known to us, e.g., [2], contains a number of recommendations concerning this problem, but all these recommendations are of a limited nature. Most promising here is the use of special converters of graphic information into digital information [4]. The present level of the technical means of computer graphics makes it possible to solve the problem of the interactive input of information into a computer, in particular with the aid of a display, and this solution then is sufficiently universal and effective.

Let us examine the possibilities of automating the finite-element procedure on the example of the solution of a nonlinear heat-conduction equation of the form

$$([\mathcal{K}]+[\mathcal{D}])\{\mathcal{L}\}+[\mathcal{H}]\frac{\partial}{\partial\tau}\{\mathcal{L}\}+\{\mathcal{F}\}=\{0\}, \tag{1}$$

where $[\mathcal{K}]$, $[\mathcal{D}]$, $[\mathcal{H}]$ are the matrices of heat conduction, additional heat conduction, and thermal capacity, respectively; $\{\mathcal{F}\}$, $\{\mathcal{L}\}$ are the column vectors of the thermal load and of the unknown values of the temperature, respectively.

In the stepped procedure in time by the Bubnov–Galerkin method, the integration of (1) reduces to the successive solution of the equations by the recurrent formula

$$\{\mathcal{L}_i\}=-[2\Delta\tau([\mathcal{K}]+[\mathcal{D}])+3[\mathcal{H}]]^{-1}[[\Delta\tau([\mathcal{K}]+[\mathcal{D}])-3[\mathcal{H}]]\{\mathcal{L}_{i-1}\}+3\Delta\tau\{\mathcal{F}\}], \tag{2}$$

where $\Delta\tau$ is the time of the change in temperature at the nodes of the coating from $\{\mathcal{L}_{i-1}\}$ to $\{\mathcal{L}_i\}$.

The dependences of the thermophysical characteristics of the material on the temperature and the current porosity impose a strong limitation on the selection of $\Delta\tau$. By correspondingly reducing the time step it is possible to linearize the regularities of these characteristics and to select their values from the preceding temporary layer [5].

We realize the generation of (2) with the following main assumptions. We examine a plane quasi-steady-state heat-conduction problem. The region Ω whose temperature field is

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TABLE 1. Recurrent Dependences for Generating the Matrix of Indices

Code	Local indexing $\{\delta\}$	Coordinate coefficients in (3)	Global indexing
2	1 2 3	$b_1=0$ $c_1=-h_1$ $b_2=h_2$ $c_2=h_1$ $b_3=-h_2$ $c_3=0$	$\mathscr{L}_{K1}=(k+3)/2+m+n_K$ $\mathscr{L}_{K2}=\mathscr{L}_{K1}-1$ $\mathscr{L}_{K3}=\mathscr{L}_{K2}-m-1$
3	3 1 2	$b_1=-h_2$ $c_1=-h_1$ $b_2=h_2$ $c_2=0$ $b_3=0$ $c_3=h_1$	$\mathscr{L}_{K1}=k/2+n_K$ $\mathscr{L}_{K2}=\mathscr{L}_{K1}+m+1$ $\mathscr{L}_{K3}=\mathscr{L}_{K1}-1$
4	3 2 1	$b_1=0$ $c_1=-h_1$ $b_2=h_2$ $c_2=0$ $b_3=-h_2$ $c_3=h_1$	$\mathscr{L}_{K1}=(k+1)/2+n_K$ $\mathscr{L}_{K2}=\mathscr{L}_{K1}+m+1$ $\mathscr{L}_{K3}=\mathscr{L}_{K1}-1$
5	3 1 2	$b_1=-h_2$ $c_1=0$ $b_2=h_2$ $c_2=-h_1$ $b_3=0$ $c_3=h_1$	$\mathscr{L}_{K1}=k/2+n_K$ $\mathscr{L}_{K2}=\mathscr{L}_{K1}+m+1$ $\mathscr{L}_{K3}=\mathscr{L}_{K2}-1$

Note: n_K, number of the column in the matrix of codes; K, number of the FE; m, number of rows in the matrix of codes; code 1 corresponds to the combined FE of the 2nd and 3rd codes.

investigated is represented by a set of finite elements (FE) in the form of regular triangles of unit thickness. Each FE has three degrees of freedom and is determined by the temperature values at the vertices of the triangle. Within the limits of an element the change in temperature is linear, and the thermophysical characteristics (λ, a) are constant. We want to point out that when there are structural and phase transformations and material is removed, the accuracy of the solution of (1) is strongly affected by a change in the geometry of the domain with time. As a rule, this makes it necessary to generate (2) in the dynamic regime and to rearrange the finite-element network.

To automate the procedure of generating (2), we limit ourselves to the spatial orientation of the FE as shown in Table 1. The matrices $[\mathscr{K}]$ and $[\mathscr{H}]$ are compiled according to the standard rules of the finite-element procedure from the submatrices $[K]$ and $[H]$. The coefficients of the submatrices are calculated in the following manner:

$$K_{ij}=\frac{\lambda}{4s}(b_ib_j+c_ic_j)\ (i,\ j=1,\ 2,\ 3), \tag{3}$$

$$\frac{a}{s\lambda}H_{ij}=\begin{cases}\frac{1}{6} & \text{for } i=j\\ \frac{1}{12} & \text{for } i\neq j\end{cases}\quad (i,\ j=1,\ 2,\ 3), \tag{4}$$

where i and j are the numbers of nodes in the local frame of reference.

The matrix $[\mathscr{D}]$ is generated for the ensemble of elements as a whole, with

$$\frac{1}{\lambda h}D_{ij}=\begin{cases}\frac{1}{3} & \text{for } i=j\\ \frac{1}{6} & \text{for } i\neq j\end{cases}\quad (i,\ j=1,\ 2), \tag{5}$$

where h is the length of that side of the FE that receives the thermal flux.

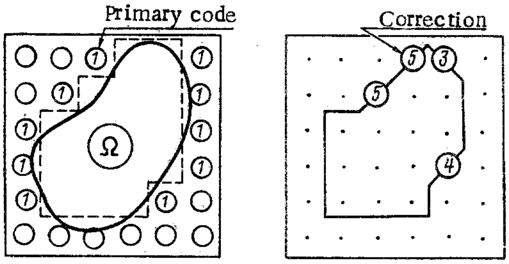

Fig. 1. Identification of the investigated region on the display screen.

The vector $\{\mathcal{F}\}$ is determined with the aid of the representation of distributed thermal load by a system of concentrated vectors acting at the nodes of the coating.

The most widespread method of computer generation of $[\mathcal{K}]$ and $[\mathcal{H}]$ from the respective submatrices is the method based on the use of the identity operator in the form of a matrix of indices (MI). The principles of the visual generation of MI are fairly well known [2] and widely used in practice.

On the example of information processing according to the geometry of the object we will describe the main principles of automatic generation of MI with the aid of interactive input on the display. For this we will briefly examine the main stages of generating MI for the domain Ω illustrated in Fig. 1.

1. On strong paper the investigated region is plotted on a scale determined by the size of the display screen. The outline is determined in such a way as to obtain a template with which Ω is identified on the display. The light matrix on the screen contains buffer memory cells, each element of which is identified with a finite element. The types of elements are identified in accordance with Table 1. Before work begins, all cells contain zeros, and the boundaries of Ω are registered according to the template in the first approximation by the homogeneous code 1. Then the boundaries are corrected in the way shown in Fig. 1.

2. As a result of the operations according to point 1 we obtain in the buffer memory of the display information on the geometry of the region in the form of a matrix of codes and a scale factor. The body of information proceeds by rows to the operational memory of the computer when it is interpreted and completed with the aid of a special subprogram. The primary codes outside Ω are removed, and the FE forming the investigated region is again assigned the codes 1. That way a matrix of the form

$$\begin{bmatrix} 0 & 0 & 0 & 5 & 3 & 0 \\ 0 & 0 & 5 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is generated which corresponds to the finite-element representation of Ω in the form of Fig. 2. Decoding it in accordance with the recurrent dependences presented in Table 1, the com-

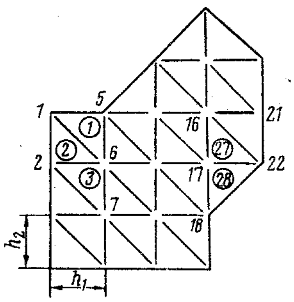

Fig. 2. Network domain with elements and node unknowns.

TABLE 2. Matrix of Indices

Elements		$\{\delta\}$		
		1	2	3
No.	code	$\{\mathscr{L}\}$		
1	2	6	5	1
2	3	2	6	1
3	2	7	6	2
⋮	⋮	⋮	⋮	⋮
27	3	17	22	16
28	4	18	22	17
⋮	⋮	⋮	⋮	⋮

puter generates the MI in Table 2; this ensures later the automatic plotting of $[\mathscr{K}]$ in the operative computer memory.

Thus we examined the main stages of the procedure enabling us to effect the automatic superposition of the network on a region of arbitrary shape in such a way that the boundaries of the region are inscribed into the line of the network. It is natural that the greater the power of resolution of the display is, the more accurate is the geometric approximation of the investigated object. Analogously, we carry out the interactive input of information with the aid of the display also for other matrices contained in (1).

According to these procedures, the present authors devised and tested a program making it possible to make maximum use of the possibilities of the display for the input of graphic information and information in the form of letters and digits in the solution of variegated heat-conduction problems by the finite-element method.

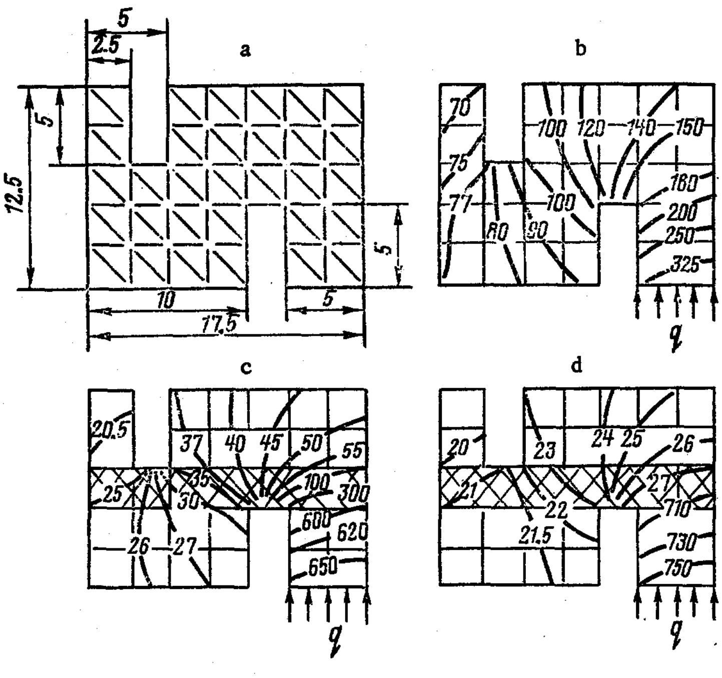

Fig. 3. Computer experiment for choosing heat insulation: a) dimensions of the structure; b) temperature field of the homogeneous medium; c, d) temperature fields with different coefficients.

In conclusion and for evaluating the efficiency of the method, we present the results of a computer experiment with the selection of heat insulation in a structure (Fig. 3a) represented by the structural model in Fig. 3. The parameters of the models are: $\theta = 1820°C$; $T_0 = 20°C$; $\alpha = 1350\ W/m^2 \cdot deg$; $\tau = 5$ sec; $\Delta\tau = 0.025$ sec. Heat insulation: steel 35 (b); porcelain (c); asbestos (d).

The temperature fields represented by isotherms were obtained on the basis of computers ES-1020 and MIR-2 using a compiling input—output generator for recoding the information from one computer to the other. The total computer time of the experiment was 30 min. An analogous solution of the problem by the implicit method of triangle with the matching method according to subregions [5] is obtained after 80 min.

NOTATION

λ, thermal conductivity; a, thermal diffusivity; s, area of the finite element; α, heat-transfer coefficient; T_0, initial wall temperature; θ, ambient temperature; τ, time.

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